RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2019

FIRST YEAR (BATCH 2019-22)

Date : 13/12/2019 **PHYSICS (Honours)**

Time : 11 am – 1 pm **Paper : II [CC2]** Full Marks : 50

Answer <u>any five</u> questions of the following : [5×10]

1. a) Express $\hat{r}$ and $\hat{\theta}$ in Cartesian co-ordinate system and prove the following identities.

   i) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

   ii) $\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$

   where $\vec{v}$ and $\vec{a}$ are velocity and acceleration respectively. [5]

   b) What is conservative force? Show that the gravitational force is conservative. Give two examples of non-conservative force. [1+3+1]

2. a) Two particles of equal mass m are connected by a massless rope of length 2r. They are isolated in space orbiting around their centre of mass at speed v. Calculate the angular momentum and kinetic energy of the system. What is the radius of gyration? [4+1]

   b) Two particles of mass m and M undergo uniform circular motion about each other at a separation R under the influence of an attractive force F. The angular velocity is ω radians per second. Show that $R = \frac{F}{\omega^2}\left(\frac{1}{m} + \frac{1}{M}\right)$ [5]

3. a) Find the centre of mass of a cricket bat with width a and b at the blade and the handle respectively and with total length *l*, the length of the handle being one-fourth of the total length. [5]

   b) A freight car of mass M contains a mass of sand m. At t = 0, a constant horizontal force F is applied in the direction of rolling and at the same time, a part at the bottom is opened to let the sand flow out at constant rate $\frac{dm}{dt}$. Find the speed of the freight car when all the sand is gone. Assume the freight car is at rest at t = 0. [5]

4. a) The interaction potential between two atoms is given by

   $$U = \varepsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right].$$

   i) Show that the radius at the potential minimum is r_o.

   ii) Find the frequency of small oscillations about equilibrium for 2 identical atoms of mass m bound to each other by the given interaction. [5]

   b) A stick of length *l* and mass M, initially upright on a frictionless table, starts falling. Find the speed of the centre of mass as a function of position, taking 'y' and θ as parameters. [Here 'y' is distance of descend of the centre of mass while θ is the angle made by the stick with the horizontal at that very instant]. [5]

5. a) Show that a shear in equivalent to simultaneous compression and extension along mutually perpendicular directions. [4]

   b) Show that the Poission's ratio of a wire is $-\frac{1}{2}$ when volume remains constant. [2]

   c) Liquid of density ρ and coefficient of viscosity η flows in stream lines through a capillary tube of radius R and length *l*. The velocity of the liquid at a distance r from the axis of the tube is

given by $v(r) = v_0\left(1 - r^2/R^2\right)$. Find (i) the kinetic energy of the liquid within the volume of the tube, (ii) the pressure difference between the ends of the tube. [2+2]

6. a) A particle is moving under the attractive inverse square law $f(r) = -k/r^2$ for $k > 0$. Show that (i) the orbit of the particle is elliptic and (ii) the square of the period of revolution of the particle is proportional to the cube of its semi-major axis. [6]

   b) Two bodies of masses M_1 and M_2 are placed with distance d apart. Show that at a point, where the gravitational field is zero, the potential is given by $V = -\frac{G}{d}\left(M_1 + M_2 + 2\sqrt{M_1 M_2}\right)$. [4]

7. a) A 1.5 kg weight hung on a vertical spring stretches it 0.4 m. The weight is then pulled down 1 m and released. Is the motion oscillatory damped, over damped or critically damped, when the damping force is numerically equal to (i) 15 times, and (ii) 7.5 times the instantaneous speed. Find the position of the weight at any time in each of the above cases. [2+4]

   b) A damped system is represented by the equation.

$$\frac{d^2x}{dt^2} + 14\frac{dx}{dt} + 49x = 0$$

   What is the nature of damping? Justify your answer. Give an example of such a damping. [3+1]

8. a) Using suitable approximation show that the equation of motion of a particle relative to an observer on Earth's surface is given by,

$\frac{d^2\vec{r}}{dt^2} = \vec{g} - 2\left(\vec{\omega} \times \vec{v}\right)$, where $\vec{\omega}$ is the angular velocity of earth. [4]

   b) If a particle is fired with initial velocity $v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ from the origin of a coordinate system fixed relative to the Earth's surface of at colatitude λ, prove that its position at any later time t will be given by

$$x = v_1 t + \omega v_2 t^2 \cos\lambda$$

$$y = v_2 t - \omega t^2\left(v_1 \cos\lambda + v_3 \sin\lambda\right) + \frac{1}{3}\omega g t^3 \sin\lambda$$

$$z = v_3 t - \frac{1}{2} g t^2 + \omega v_2 t^2 \sin\lambda$$

   neglecting terms involving ω^2. [6]

———— × ————